Z-transform right shift property: $f[n-m]u[n] \Leftrightarrow z^{-m}F[z] + z^{-m}\left(\sum_{n=1}^{m} f[-n]z^{n}\right)$

Derivation: Consider an alternative expression for the unit step, $u[n]$:

$$u[n] = \prod_{m}[n] + u[n-m]$$

where $\prod_{m}[n]$ is a rectangular function with unit magnitude and width m given by:

$$\prod_{m}[n] = u[n] - u[n-m]$$

Using the above expression for the unit step, the z-transform property can be written as:

$$\rightarrow f[n-m]u[n] = f[n-m]\prod_{m}[n] + f[n-m]u[n-m]$$

The rectangular function, $\prod_{m}[n]$ is just a sum of m time-shifted impulse functions:

$$\prod_{m}[n] = \sum_{l=0}^{m-1} \delta[n-l]$$

$$\rightarrow \therefore f[n-m]u[n] = \sum_{l=0}^{m-1} f[l-m]\delta[n-l] + f[n-m]u[n-m]$$

where the $f[l-m]$ terms in the sum are the weights of the impulse function

Writing the expression by expanding the sum:

$$\rightarrow f[n-m]u[n]$$
$$= f[-m]\delta[n] + f[1-m]\delta[n-1] + \cdots\cdots + f[-1]\delta[n-(m-1)] + f[n-m]u[n-m]$$

Taking the z-transform of both sides:

$$\rightarrow Z\{f[n-m]u[n]\} = f[-m] + f[1-m]z^{-1} + \cdots\cdots + f[-1]z^{-m+1} + z^{-m}F[z]$$

factor out z^{-m} from all the summed terms:

$$= z^{-m}\left(\sum_{n=1}^{m} f[-n]\,z^{n}\right) + z^{-m}F[z]$$